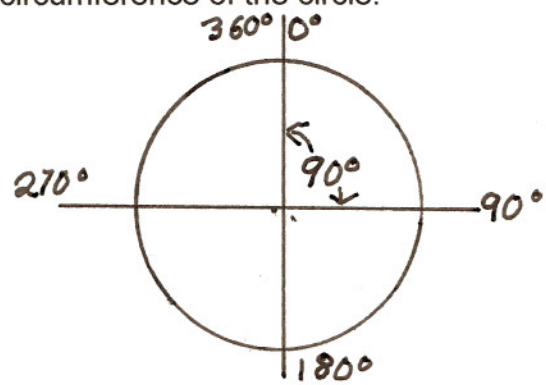
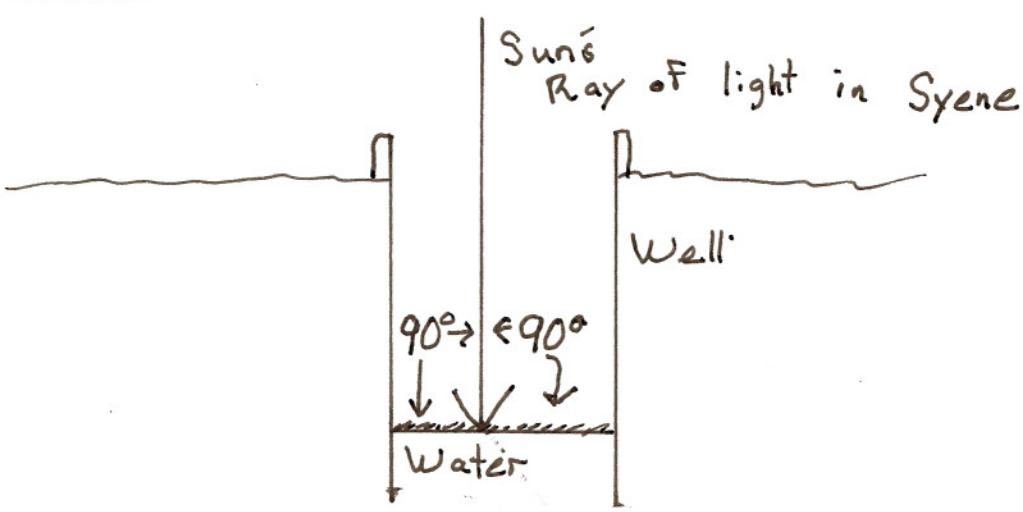
Eratosthenes and the Circumference of the Earth

Eratosthenes of Syene (275-194 B.C.) successfully measured the circumference of the earth by knowing a few simple facts and putting them together. He knew:

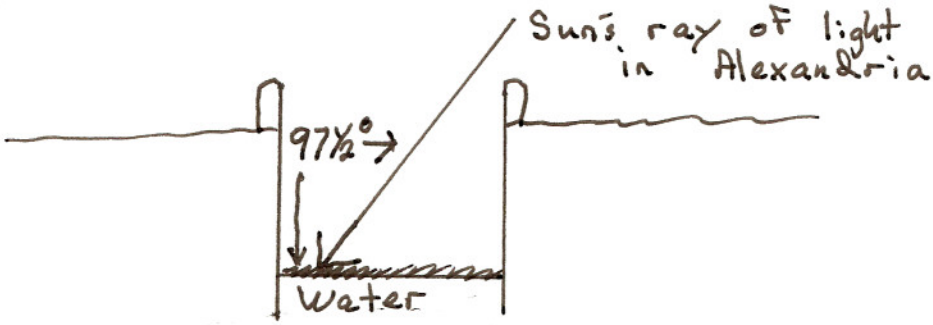
1) That the earth was a sphere, not flat or rectangular as earlier Greeks had postulated. Only a spherical earth could explain the round earth shadow on the moon's surface during a lunar eclipse.
2) Eratosthenes knew the distance between Alexandria and Syene in Hellenistic Egypt as approximately 500 miles.
3) He also knew that the sun set in Alexandria and Syene at almost the same time every evening. That confirmed that the two cities rested on what we would call today, the same line of longitude. Put another way, the cities were directly north and south of each other but neither one was further east or west of the other.
4) Finally he understood that you can divide a circle into 360 degrees. Each degree represents not just an angle between two lines that travel from the center of the circle to its outer surface or circumference. Each degree also represents a specific distance between those two lines where they intersect with the surface or circumference of the circle.



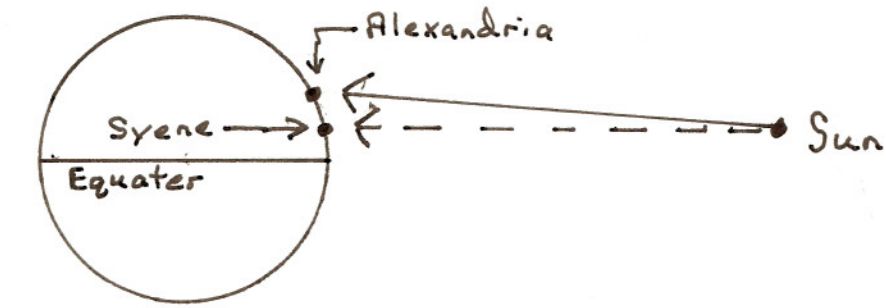
The legend began when Eratosthenes was in the city of Syene on the longest day of the year, June 21st. He looked into a deep well at noon and noticed that the sun's rays directly reflected off the water below. Usually the sun's rays never made it past the inside walls of any well so Eratosthenes started to think. He figured correctly that the sun must be directly overhead for its rays to reflect off the water at the bottom. Put another way, the sun's rays, which formed a straight line from the flat surface of the water to the sun, also formed one side of a perfect 90 degree right triangle. The bottom of the right triangle was the flat surface of the water.



The next year Eratosthenes traveled 500 miles north to Alexandria on June 21st and at 12 noon looked into another deep well. The sunlight reflected mostly off the wall of this well and only slightly touched the surface of the water. Eratosthenes calculated that unlike the well in Syene, the sun here was not directly overhead. Thus the sun's rays entered the well at a slight deviation from the 90 degree angle in Syene. Eratosthenes measured the angle at 7 ½ degrees difference from Syene. Here the sun formed an angle of 97 ½ degrees with the level surface of the water at the bottom of the well.



If we depict the earth as a circle we can start to see what Eratosthenes understood. The below picture is not to scale. The sun is actually much larger than the earth. However since it is so far away, 93 million miles, it is accurate to depict it as a small, fixed point of light just like any star. Notice how the sun's rays shine directly over Syene while hitting Alexandria at an angle.



Eratosthenes used a simple ratio to determine the circumference of the earth. He knew that there was 360 degrees in a circle. He measured the difference in angles of sunlight hitting the earth's surface in Alexandria and Syene as 7 ½ degrees. If we divide 360 degrees by 7 ½ degrees we come up with 50. Put another way, 7 ½ degrees represents 1/50th of the circumference of the earth. Since that also corresponded to a known distance of 500 miles, the distance between Alexandria and Syene, then 500 miles times 50 must equal the circumference of the earth. Eratosthenes technique was brilliantly simple and yielded an amazingly accurate figure. The actual circumference of the earth at from north to south today, is 24,859 miles.

Tim Brockett
timbrockett@sprintmail.com